Discrete optimisation - Tutorial #1

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On the next page is the Simplex algorithm as seen in the first lecture.

1. Please review it for comprehension
2. Run the first exercise *by hand*.
3. Implement the algorithm in the language of your choice. I recommend Python or R. An initial feasible basis is given as argument.
4. Verify your hand-computed solution.
5. Complete the remaining solution.

The Simplex algorithm for a minimization

Initialisation Let $B = B^0$ be a feasible basis solution (FBS). We assume B^{-1} exists and $B^{-1}b \geq 0$.

While The solution is non-optimal and bounded, do :

1. *Compute*
   —
$$\bar{b} = B^{-1}b \text{ (Feasible Basis Solution)} \qquad (1)$$

   —
$$\bar{c}_e^T = c_e^T - c_b^T B^{-1}E \text{ (Reduced costs)}$$

2. — *If $\bar{c}_e^T \geq 0$* : Optimal solution The solution is provided from equation (1) above.
   — *Else* choose x_l such that $\bar{c}_l < 0$

3. *Compute*
$$P = B^{-1}A_l \ (A_l \text{ is column } l \text{ of } A)$$

4. — *If : $\forall i, P_i \leq 0$* : unbounded solution
   — *Else* :

$$x_l = \min_{k/P_k>0} \left\{ \frac{\bar{b}_k}{P_k} \right\} \qquad (1)$$

$$j = \operatorname{argmin}_{k/P_k>0} \left\{ \frac{\bar{b}_k}{P_k} \right\} \qquad (2)$$

   — Replace variable in position j in the basis by x_i.

End while

1 Problems

1.1 Problem 1

A company makes products I, II and III from some some resources. Here are the resources consumed for each unit of product produced :

Product	I	II	III	Resource availability
Machine time	2	3	1	10
Primary materials	1	4	3	15

Profits values from each product are respectively 6, 4 and 5.

a) Formulate the problem as an LP, to maximize the profit.

b) Put in standard form.

c) Solve by the simplex algorithm.

1.2 Problem 2

A company produces 3 types of products (A, B, and C), and can sell them in unlimited quantities at the following prices : A : 10 euros, B : 56 euros, C : 100 euros
Production constraints are the following :

— To produce one unit of A requires 1h of work.
— To produce one unit of B requires 2h of work + two units of A.
— To produce one unit of C requires 3h of work + 1 unit of B.
— The company only has 35h of work at its disposal only.

a) Formulate the problem as an LP
b) Put in standard form
c) Solve the problem using the simplex algorithm